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Nonlinear surface polaritons in 2D electron system in high magnetic field

N.N. Beletskii[1] and Y.V. Bludov[a]

Usikov Institute for radiophysics and electronics, NAS of Ukraine. 12, Acad. Proscura st., Kharkov, 61085, Ukraine

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Abstract. The paper deals with the theoretical investigation of nonlinear surface polaritons (NSP) in isolated two-dimensional electron system (2DES) arranged at the interface between linear and nonlinear media and placed into the external quantizing magnetic field directed perpendicularly to 2DES. We consider that nonlinear medium dielectric permeability depends upon the tangential component of electric field only. It is shown that under the integer quantum Hall effect conditions all NSP characteristics are represented by the quantized values. It is found that the NSP spectrum contains two NSP modes – high-frequency and low-frequency ones. It is shown that the NSP can exist only in the case where the value of tangential component of electric field at the interface is less than a certain critical value. It is found that the resonant interaction between the NSP high-frequency mode and surface polariton mode occurs in the vicinity of the cyclotron resonance subharmonic.

PACS. 73.20.Mf Collective excitations (including excitons, polarons, plasmons and other charge-density excitations)

1 Introduction

Surface polaritons (SP) in nonlinear media possess some new features in comparison with SP in linear media. For example, an excitation of SP becomes possible without using prisms and periodical structures. It can occur when the finite wave train falls directly upon the interface [1].

The nonlinear surface polaritons (NSP) were earlier investigated at a single interface [2] and also in a superlattice [3]. The NSP at the interface between linear and nonlinear media were studied theoretically in [2] in the case of quadratic dependence of nonlinear medium dielectric permeability upon the tangential component of TM-wave electric field. In [2] it was shown that depending on the relation between signs of dielectric constants of contacting media two types of NSP can exist at the interface. In the case where signs of the above-mentioned dielectric constants are different, the electric field of NSP decreases monotonically when the distance from the interface increases. At the same time in the case where the signs of dielectric constants coincide, the NSP electric field has a maximum in the region occupied by a nonlinear medium.

It is known that the SP spectrum essentially changes when the two-dimensional electron system (2DES) is placed at the interface [4,5]. It should be noticed that linear SP in the single 2DES were studied theoretically both in the absence [4] and in the presence of external quantizing [5] magnetic field.

This paper deals with the theoretical investigation of NSP in single 2DES, placed at the plane $z = 0$. The external quantizing magnetic field is directed perpendicularly to 2DES along the axis z. We consider that the external magnetic field is high enough to cause the phenomenon of integer quantum Hall effect (IQHE) in 2DES. We suppose that half-space $z < 0$ is occupied by an isotropic linear medium with the dielectric constant ε_1, and half-space $z > 0$ is occupied by a nonlinear medium. Following [2] we suppose that the nonlinear medium dielectric tensor possesses only diagonal components, which depend upon the electric field component E_x only, *i.e.*:

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon + \rho|E_x|^2, \varepsilon_{zz} = \varepsilon_0. \quad (1)$$

We use such an exotic model of the dielectric tensor to obtain the analytical expression for NSP dispersion relation. Speaking strictly, there are no crystallographic classes for which such a form of dielectric tensor arises from the symmetry considerations. However, the cases are possible when the distinction from that model is neglible [6].

2 Dispersion relation

Supposing that the electromagnetic field depends upon time and coordinate as $\mathbf{E}, \mathbf{H} \sim \exp(\mathrm{i}\,[kx - \omega t])$ (here k and ω are the wavenumber and the frequency of wave,

[a] e-mail: bludov@ire.kharkov.ua

correspondingly), the Maxwell equations for complex amplitudes of electromagnetic field components can be written as

$$\frac{\mathrm{d}E_x^+}{\mathrm{d}z} - \mathrm{i}kE_z^+ = \frac{\mathrm{i}\omega}{c}H_y^+, \quad -\frac{\mathrm{d}H_y^+}{\mathrm{d}z} = -\frac{\mathrm{i}\omega}{c}\varepsilon_{xx}E_x^+, \quad (2)$$
$$kH_y^+ = -\frac{\omega}{c}\varepsilon_{zz}E_z^+,$$

$$-\frac{\mathrm{d}E_y^+}{\mathrm{d}z} = \frac{\mathrm{i}\omega}{c}H_x^+, \quad \frac{\mathrm{d}H_x^+}{\mathrm{d}z} - \mathrm{i}kH_z = -\frac{\mathrm{i}\omega}{c}\varepsilon_{yy}E_y^+, \quad (3)$$
$$kE_y^+ = \frac{\omega}{c}H_z^+,$$

$$\frac{\mathrm{d}E_x^-}{\mathrm{d}z} - \mathrm{i}kE_z^- = \frac{\mathrm{i}\omega}{c}H_y^-, \quad -\frac{\mathrm{d}H_y^-}{\mathrm{d}z} = -\frac{\mathrm{i}\omega}{c}\varepsilon_1 E_x^-, \quad (4)$$
$$kH_y^- = -\frac{\omega}{c}\varepsilon_1 E_z^-,$$

$$-\frac{\mathrm{d}E_y^-}{\mathrm{d}z} = \frac{\mathrm{i}\omega}{c}H_x^-, \quad \frac{\mathrm{d}H_x^-}{\mathrm{d}z} - \mathrm{i}kH_z^- = -\frac{\mathrm{i}\omega}{c}\varepsilon_1 E_y^-, \quad (5)$$
$$kE_y^- = \frac{\omega}{c}H_z^-.$$

Here sign "+" corresponds to the electromagnetic field components in the nonlinear medium, and sign "−" corresponds to the components in the linear one.

Eliminating H_y^+ and E_z^+ from the system of equations (2) and taking into account formulas (1), we obtain the differential equation for the electric field component E_x^+ in the nonlinear medium:

$$\frac{\mathrm{d}^2E_x^+}{\mathrm{d}z^2} - \frac{p_0^2}{\varepsilon_0}\left[\varepsilon + \rho(E_x^+)^2\right]E_x^+ = 0, \quad (6)$$

where $p_0^2 = k^2 - \omega^2\varepsilon_{zz}/c^2$. We consider the case, when $\rho < 0$. Under conditions $E_x^+(\infty) \to 0, \left.\frac{\mathrm{d}E_x^+}{\mathrm{d}z}\right|_{z\to\infty} \to 0$, the solution of equation (6) takes the form:

$$E_x^+(k,\omega,z) = \sqrt{2\varepsilon/|\rho|}\cosh^{-1} p_0\sqrt{\frac{\varepsilon}{\varepsilon_0}}(z - z_0). \quad (7)$$

Here z_0 is the integration constant.

Eliminating H_x^+ and H_z^+ from the system of equations (3) and taking into account formula (7) for E_x^+, we obtain the differential equation for the electric field component E_y^+:

$$\frac{\mathrm{d}^2E_y^+}{\mathrm{d}z^2} - \left(p^2 + 2\frac{\omega^2\varepsilon}{c^2}\cosh^{-2} p_0\sqrt{\frac{\varepsilon}{\varepsilon_0}}(z-z_0)\right)E_y^+ = 0, \quad (8)$$

where $p^2 = k^2 - \omega^2\varepsilon/c^2$. The solution of equation (8), which satisfy condition $E_y^+(\infty) \to 0$, can be obtained using the JWKB method in the form:

$$E_y^+(z) = E_y^+(0)\exp\left\{-pz - \frac{\omega^2}{c^2p}\frac{\sqrt{\varepsilon_0\varepsilon}}{p_0}\right. \quad (9)$$
$$\left.\times\left[\tanh\left(p_0\sqrt{\frac{\varepsilon}{\varepsilon_0}}(z-z_0)\right) + \tanh\left(p_0\sqrt{\frac{\varepsilon}{\varepsilon_0}}z_0\right)\right]\right\}.$$

It should be emphasized that the JWKB-solution of equation (8) is true only when $p \gg 1$.

The solutions of Maxwell equations for tangential components of the SP electromagnetic field in the linear medium under conditions $\mathbf{E}^-, \mathbf{H}^-|_{z\to-\infty} \to 0$ can be written as

$$E_x^-(k,\omega,z) = E_x^-(0)\exp(p_1 z), \quad (10)$$
$$E_y^-(k,\omega,z) = E_y^-(0)\exp(p_1 z), \quad (11)$$

where $p_1^2 = k^2 - \omega^2\varepsilon_1/c^2$. To obtain the dispersion relation, which describes the NSP propagation, we use standard boundary conditions. We consider that tangential components of electric field are continuous across the interface $z = 0$, and tangential components of magnetic field are discontinuous across the interface due to the presence of currents in 2DES. In other words,

$$H_x^+ - H_x^- = (4\pi/c)j_{ys} = (4\pi/c)(\sigma_{yx}E_x + \sigma_{yy}E_y),$$
$$H_y^+ - H_y^- = -(4\pi/c)j_{xs} = -(4\pi/c)(\sigma_{xx}E_x + \sigma_{xy}E_y).$$

Applying such boundary conditions at the interface $z = 0$, after some algebra we obtain the dispersion relation in the form:

$$\left\{\frac{\mathrm{i}\omega}{c}\left(\frac{(\varepsilon_0\varepsilon)^{1/2}}{p_0}T - \frac{\varepsilon_1}{p_1}\right) + \frac{4\pi}{c}\sigma_{xx}\right\}\left\{\frac{\mathrm{i}c}{\omega}\left[p + p_1\right.\right.$$
$$\left.\left.+\frac{\omega^2\varepsilon}{pc^2}(1-T^2)\right] + \frac{4\pi}{c}\sigma_{yy}\right\} - \left(\frac{4\pi}{c}\right)^2\sigma_{xy}\sigma_{yx} = 0, \quad (12)$$

where $T = \tanh(p_0\sqrt{\varepsilon/\varepsilon_0}z_0) = \pm\sqrt{1 - |\rho|E_x^2(0)/(2\varepsilon)}$, $E_x(0)=E_x^+(0)=E_x^-(0)$. Notice that in the case where the nonlinearity is absent ($\rho = 0$) and $\varepsilon_0 = \varepsilon$, the equation (12) coincides with the dispersion relation for linear SP [6]. For the analysis of the NSP dispersion properties we use the formulas for $\sigma_{\alpha\beta}(k,\omega)$ under the IQHE conditions, obtained in [5]:

$$\sigma_{xx}(k,\omega) = \sigma_{yy}(k,\omega) = \frac{2e^2}{h}\frac{\aleph\gamma}{1+\gamma^2}\left[1 - C(\gamma^2+2)\right], \quad (13)$$

$$\sigma_{xy}(k,\omega) = -\sigma_{yx}(k,\omega) = \frac{2e^2}{h}\frac{\aleph}{1+\gamma^2}\left[1 + 3C\gamma^2\right]. \quad (14)$$

Here $C = \frac{(k\ell)^2}{\gamma^2(\gamma^2+4)}\left(1 + \frac{\aleph}{2}\right)$, $\gamma = (\nu - \mathrm{i}\omega)/\Omega$, $\Omega = eB/m^*c$ is the frequency of the cyclotron resonance (CR), e, m^*, ν are the charge, the effective mass and the momentum relaxation frequency of electrons, $\ell = (c\hbar/eB)^{1/2}$ is the magnetic length, $\aleph = \pi\ell^2 n$ is the Landau-level filling factor, n is the two-dimensional density of electrons in 2DES.

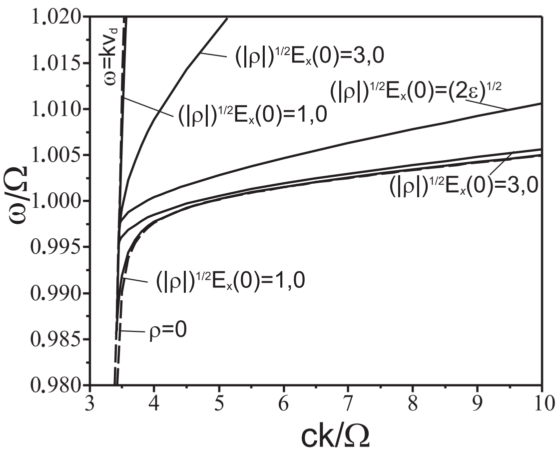

Fig. 1. NSP dispersion curves in 2DES in the case when $\varepsilon = \varepsilon_0 = \varepsilon_1 = 12$, $\aleph = 1$.

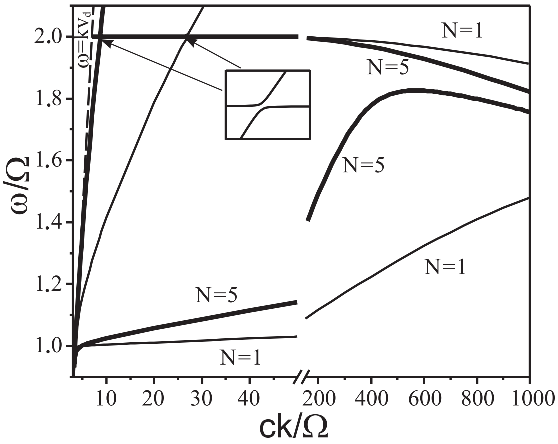

Fig. 2. NSP dispersion curves in 2DES in the case when $\varepsilon = \varepsilon_0 = \varepsilon_1 = 12$, $(|\rho|)^{1/2} E_x(0) = 3.0$ and for two values of Landau-level filling factor: $\aleph = 5$ (bold solid curves) and $\aleph = 1$ (thin solid curves).

3 Numerical results

Figure 1 presents the dispersion curves, describing the NSP propagation in 2DES, for different values of x-component of the electric field at the interface $E_x(0)$. These dispersion curves were obtained by the numerical solution of equation (12). The x-axis gives the dimensionless wavenumber ck/Ω, and the y-axis gives the dimensionless frequency ω/Ω. The SP dispersion curve in the case, where 2DES is placed at the boundary of two linear media with the same parameters, is depicted by a dashed line $\rho = 0$. The "light line" $\omega = k\, v_d$ is depicted in Figure 1 by a dashed line. Here $v_d = c/\sqrt{\varepsilon}$ is the phase velocity of light in dielectric with dielectric constant ε. As the model of 2DES we used a heterostructure $GaAs/Al_xGa_{1-x}As$ with the effective mass of the electrons $m^* = 0.068 m_0$ (m_0 is the mass of free electron) and with the dielectric constant $\varepsilon = \varepsilon_0 = \varepsilon_1 = 12$. The dissipation of electrons in 2DES was not taken into account ($\nu = 0$).

Notice that it follows from the formula (7) that NSP can exist only as $E_x(0) \le E_x^{max} = \sqrt{2\varepsilon/|\rho|}$ (in opposite case the parameter z_0 will not be real value). As seen from Figure 1, when $\rho < 0$, the NSP dispersion curves exist in the higher-frequency region in comparison with the SP dispersion curve in the linear case. That fact can be explained like this: it follows from formulas (1) that if $\rho < 0$ and $E_x \neq 0$, then $\varepsilon_{xx} = \varepsilon_{yy} < \varepsilon$. So, in that case nonlinearity leads to the decrease of the effective value of the dielectric constant. In it's turn it causes shifting of the NSP dispersion curves to the higher-frequency region. It should be noticed that two modes of NSP correspond to each value of $E_x(0) < E_x^{max}$. Low-frequency NSP mode corresponds to the case, where $z_0 < 0$. It should be emphasized that the dispersion curves, which correspond to that low-frequency NSP mode, have the end-points of the spectrum $p = 0$, lying on the light line. As distinct from the linear case the end-points of these dispersion curves lie in the vicinity of the CR. At the same time the dispersion curves corresponding to low-frequency NSP modes at high values of wavenumber practically coincides with the SP dispersion curves in the linear case. The high-frequency NSP mode corresponds to the case, where $z_0 > 0$. Notice that this NSP mode have no analogue in the linear case. As seen from Figure 1, the spectrum end-points of these dispersion curves coincide with the spectrum end-points of corresponding low-frequency NSP modes. With an increase of $E_x(0)$ the end-points of the NSP spectrum shift to the higher-frequency region. Also with the increase of $E_x(0)$ dispersion curves, corresponding to high-frequency and low-frequency modes, gradually draw together and when $E_x(0) = E_x^{max}$ two NSP modes blend into one mode (in this case $z_0 = 0$). It should be emphasized that as the value of external quantizing magnetic field varies all the NSP characteristics undergo the variation in discrete steps due to step-like dependence of $\aleph(B)$.

Now we consider the peculiarities of NSP dispersion curves in a wider range of frequencies and wavenumbers. Figure 2 presents the NSP dispersion curves for the fixed value $E_x(0)$ and for two values of $\aleph$. As Figure 2 shows, the low-frequency NSP mode at high values of k is characterized by anomalous dispersion. At the same time with the increase of $\aleph$ the point at which the character of NSP dispersion changes from normal to anomalous shifts to the region of smaller values of k (to the longer-wave region). At the same time the resonant interaction between the high-frequency NSP mode and the SP mode which exists near the CR subharmonic frequency occurs in the vicinity of that CR subharmonic frequency (that above-mentioned SP mode was described in [5]). It should be emphasized that with the increase of $\aleph$ the point of resonant interaction shifts to the longer-wave region. At the same time at the point of resonant interaction the character of high-frequency NSP mode dispersion tends to change: the normal dispersion transforms into anomalous.

We consider now the spatial pattern of the NSP mode electromagnetic field. Figure 3 presents tangential components of the NSP low-frequency mode electric field in

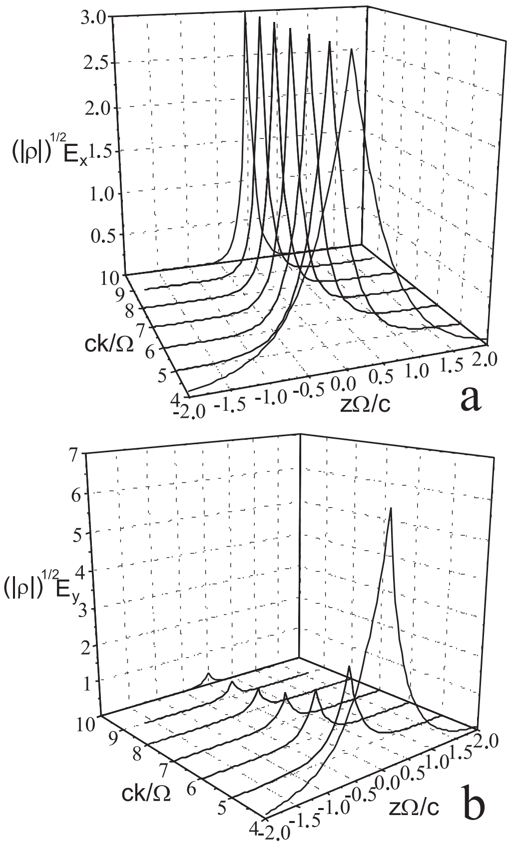

Fig. 3. Components of electric field E_x (a) and E_y (b) of NSP low-frequency mode when $(|\rho|)^{1/2}E_x(0) = 3.0$, $\aleph = 1$.

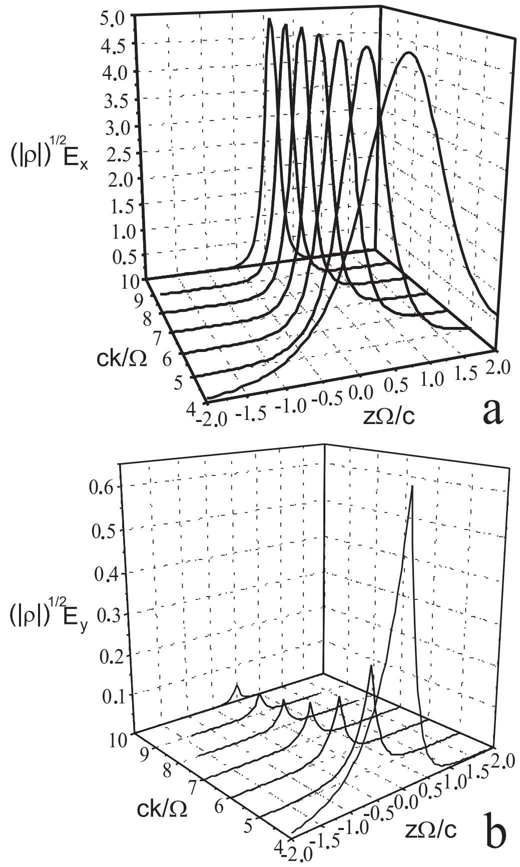

Fig. 4. Components of electric field E_x (a) and E_y (b) of NSP high-frequency mode when $(|\rho|)^{1/2}E_x(0) = 3.0$, $\aleph = 1$.

the case of fixed value of $E_x(0)$. Horizontal axes give the dimensionless wavenumber ck/Ω and dimensionless z-coordinate $\Omega z/c$. Vertical axes give the dimensionless electric fields $(|\rho|)^{1/2}E_x$ (Fig. 3a) and $(|\rho|)^{1/2}E_y$ (Fig. 3b). As seen from Figures 3a, b, magnitudes of tangential components $E_x(z)$ and $E_y(z)$ of the NSP low-frequency mode electric field decrease monotonically when the distance from the interface $z = 0$ increases. At the same time the higher the value of wavenumber k the more drastic is decrease of the electric field components $E_x(z)$, $E_y(z)$ when the distance from interface increases. It should be noted that when the value $E_x(0)$ is fixed, the value $E_y(0)$ decreases gradually with an increase of the wavenumber (Fig. 3b).

Figure 4 presents the spatial pattern of the NSP high-frequency mode electric field tangential components at the same parameters. Notice that in that case the value of $E_x(z)$ behaves as follows. When $z < 0$, value of $E_x(z)$ decreases with the increase of distance from the interface (as in the case of low-frequency mode). However at $z > 0$ the value of $E_x(z)$ possesses it's maximum E_x^{max} at point $z = z_0$. It should be emphasized that the depth of the electric field localization z_0 decreases as the wavenumber increases. Notice that the y-component of the electric field $E_y(z)$ of the high-frequency mode decreases monotonically with an increase of distance from the interface (as in the case of the NSP low-frequency mode). At the same time the value of $E_y(0)$ decreases gradually with the increase of wavenumber k.

4 Conclusion

We have calculated the spectrum of NSP in isolated 2DES placed into the quantizing magnetic field. We predicted simultaneous existence of two NSP modes and resonance interaction between the NSP high-frequency mode and the mode of SP existing in the vicinity of CR subharmonic.

References

1. D. Marcuse, Appl. Opt. **19**, 3130 (1980)
2. V.M. Agranovich, V.S. Babichenko, V.Y. Chernyak, Pis'ma v ZhETF **32**, 532 (1980) (in Russian)
3. A. Modinos, N. Stefanou, Phys. Rev. B **54**, 16452 (1996)
4. F. Stern, Phys. Rev. B. **18**, 546 (1967)
5. I.E. Aronov, N.N. Beletskii, J. Phys. Cond. Matt. **8**, 4919 (1996)
6. N.L. Dmitruk, V.G. Litovchenko, V.L. Strizhevskii, *Surface polaritons in semiconductors and dielectrics* (Naukova Dumka, 1989), 376 pp. (in Russian)